GaAs MESFET Characterization Using Least Squares Approximation by Rational Functions

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***Abstract*— We propose a new method to characterize active devices such as the FET by describing S-parameters with a set of rational functions of angular frequency. The set of rational functions is uniquely determined by only 27 coefficients, while the conventional method using tabulated S-parameters requires 8 times the number of sampling points (a typical case might require 404 data points in floating point notation). This drastically reduces the database size required to give adequate information for circuit design.**

We also describe a method for determining the equivalent circuit. Since the equivalent circuit is determined from the set of rational functions, no additional measurements are needed to determine extrinsic elements. In conventional methods, selection of initial values affects the final results. In our method, reliable initial values are extracted from the rational functions' coefficients.

The calculated S-parameters of three GaAs MESFET's having different gate widths agree closely with those measured by wafer-probe.

I. Introduction

MICROWAVE CAD requires device models with better accuracy, especially for active devices [1], [2]. The equivalent circuit approach has commonly been used to characterize active devices. In these models, however, the value of each equivalent circuit element has to be determined by rather time-consuming iteration. Berroth *et al.* determine the element values of their equivalent circuit by using a concept of cold and hot modeling instead of iteration [3], [4], but this method requires additional measurements [3]–[7] to determine extrinsic elements.

In this paper, we describe a method for determining a set of rational functions of angular frequency ω which approximates the S-parameters of the GaAs MESFET. The set of rational functions approximates measured S-parameters with excellent accuracy. The set of rational functions, as well as measured S-parameters, can be used instead of the equivalent circuit for circuit design. The set of rational functions requires much less memory than tabulated S-parameters.

This paper also describes a method for determining the equivalent circuit using the set of rational functions. Initial values of the intrinsic elements in the equivalent circuit are extracted from the coefficients of the rational functions. The extracted initial values are close to the final values, guaranteeing rapid convergence of iteration. Moreover, extraction of initial values improves accuracy of the resulting equivalent circuit, because, as has been pointed out by Dambrine *et al.* [5], selection of initial values affects the final results.

Calculated S-parameters of three GaAs MESFET's having different gate widths agree closely with those measured by wafer-probe, as will be described below.

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II. Equivalent Circuit

Fig. 1 shows the well-known equivalent circuit of the GaAs MESFET obtained from RF wafer probe measurements. We can describe the intrinsic part of the equivalent circuit by using Y-parameters, and the extrinsic part by Z-parameters, as follows:

$$Y_{\text{intrinsic}} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} \dfrac{j\omega C_{gs}}{1+j\omega C_{gs}R_i} + j\omega C_{gd} & -j\omega C_{gd} \\ g_m e^{-j\omega\tau} - j\omega C_{gd} & \dfrac{1}{R_{ds}} + j\omega(C_{gd}+C_{ds}) \end{bmatrix} \quad (1)$$

$$Z_{\text{extrinsic}} = \begin{bmatrix} Z_g + Z_s & Z_s \\ Z_s & Z_d + Z_s \end{bmatrix} \quad (2)$$

where

$$Z_g = R_g + j\omega L_g$$
$$Z_d = R_d + j\omega L_d$$
$$Z_s = R_s + j\omega L_s. \quad (3)$$

Then, the total of the Z-parameters is given as the sum of $Z_{\text{extrinsic}}$ and the inverse of $Y_{\text{intrinsic}}$, that is:

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} Z_g + Z_s & Z_s \\ Z_s & Z_d + Z_s \end{bmatrix} + \frac{1}{\Delta}\begin{bmatrix} Y_{22} & -Y_{12} \\ -Y_{21} & Y_{11} \end{bmatrix} \quad . \quad (4)$$

$$\Delta = \det(Y_{\text{intrinsic}}). \quad (5)$$

Using each element of the above Z matrix and $Z_0 = 50\,\Omega$, the S-parameters become

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \begin{bmatrix} 1 - \dfrac{2(Z_{22}+Z_0)Z_0}{P} & \dfrac{2Z_{12}Z_0}{P} \\ \dfrac{2Z_{21}Z_0}{P} & 1 - \dfrac{2(Z_{11}+Z_0)Z_0}{P} \end{bmatrix} \quad (6)$$

$$P = (Z_{11}+Z_0)(Z_{22}+Z_0) - Z_{12}Z_{21}. \quad (7)$$

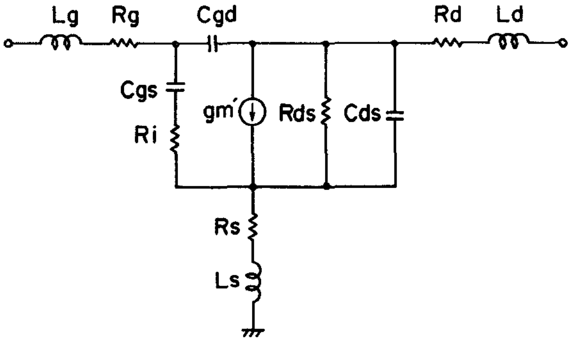

Fig. 1. Equivalent circuit of GaAs MESFET $g_m' = g_m \times \exp(-j\omega\tau)$.

If we expand the $\exp(-j\omega\tau)$ term as a power series of the angular frequency ω, the S-parameters can be expressed as a set of rational functions of ω.

III. Least Squares Approximation by Rational Functions

Suppose we have measured data of a certain transfer function $F(\omega)$ which is considered to be a rational function of angular frequency ω:

$$F(\omega_i) = \frac{\sum_{k=0}^{m} b_k (j\omega_i)^k}{\sum_{k=0}^{n} a_k (j\omega_i)^k} \tag{8}$$

where ω_i is a sampling point in the angular frequency. At least one coefficient, a_h, of the denominator is nonzero and the rational function is not changed if all the coefficients are divided by a_h. This is equivalent to assuming that a_h is equal to one.

Deviation from the measured value, expressed as $\hat{F}(\omega_i)$, is

$$\epsilon_i = \hat{F}(\omega_i) - \frac{\sum_{k=0}^{m} b_k (j\omega_i)^k}{(j\omega_i)^h + \sum_{\substack{k=0 \\ k \neq h}}^{n} a_k (j\omega_i)^k}. \tag{9}$$

Multiplying the denominator and taking the sum of the squared absolute value through all data points, the error function becomes

$$\epsilon = \sum_{i=0}^{N} \left| \left\{ (j\omega_i)^h + \sum_{\substack{k=0 \\ k \neq h}}^{n} a_k (j\omega_i)^k \right\} \cdot \hat{F}(\omega_i) - \sum_{k=0}^{n} b_k (j\omega_i)^k \right|^2. \tag{10}$$

The coefficients $\{a_k\}$ and $\{b_k\}$ which minimize ϵ can be obtained by solving the following simultaneous equations:

$$\frac{\partial \epsilon}{\partial a_p} = 0 \qquad \frac{\partial \epsilon}{\partial b_q} = 0. \tag{11}$$

Rewriting the (11) in matrix form:

$$\begin{bmatrix} \begin{bmatrix} \alpha_{00} & & \alpha_{0n} \\ & \ddots & \\ \alpha_{n0} & & \alpha_{nn} \end{bmatrix} & \begin{bmatrix} \beta_{00} & & \beta_{0m} \\ & \ddots & \\ \beta_{n0} & & \beta_{nm} \end{bmatrix} \\ \begin{bmatrix} \gamma_{00} & & \gamma_{0n} \\ & \ddots & \\ \gamma_{m0} & & \gamma_{mn} \end{bmatrix} & \begin{bmatrix} \delta_{00} & & \delta_{0m} \\ & \ddots & \\ \delta_{m0} & & \delta_{mm} \end{bmatrix} \end{bmatrix} \begin{bmatrix} \begin{bmatrix} a_0 \\ \vdots \\ a_n \end{bmatrix} \\ \begin{bmatrix} b_0 \\ \vdots \\ b_m \end{bmatrix} \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} c_0 \\ \vdots \\ c_n \end{bmatrix} \\ \begin{bmatrix} d_0 \\ \vdots \\ d_m \end{bmatrix} \end{bmatrix}. \tag{12}$$

The individual elements can be written as follows:

$$\alpha_{pq} = \sum_{i=1}^{N} \{(-j\omega_i)^p (j\omega_i)^q + (j\omega_i)^p (j\omega_i)^q\} \hat{F}(\omega_i) \hat{F}^*(\omega_i)$$

$$\beta_{pq} = \sum_{i=1}^{N} \{(-j\omega_i)^p (j\omega_i)^q \hat{F}^*(\omega_i) + (j\omega_i)^p (-j\omega_i)^q \hat{F}(\omega_i)\}$$

$$\gamma_{pq} = \sum_{i=1}^{N} \{(-j\omega_i)^p (j\omega_i)^q \hat{F}(\omega_i) + (j\omega_i)^p (-j\omega_i)^q \hat{F}^*(\omega_i)\}$$

$$\delta_{pq} = -\sum_{i=1}^{N} \{(-j\omega_i)^p (j\omega_i)^q + (j\omega_i)^p (-j\omega_i)^q\} \tag{13a}$$

$$c_p = \sum_{i=1}^{N} \{(-j\omega_i)^p (j\omega_i)^h \hat{F}(\omega_i) + (j\omega_i)^p (-j\omega_i)^h \hat{F}^*(\omega_i)\}$$

$$d_p = \sum_{i=1}^{N} \{(-j\omega_i)^p (j\omega_i)^h + (j\omega_i)^p (-j\omega_i)^h\} \hat{F}(\omega_i) \hat{F}^*(\omega_i) \tag{13b}$$

We can now determine each coefficient by solving (12).

IV. Least Squares Approximation of S-Parameters by Rational Functions

To apply the above method to the S-parameters, we defined a set of rational functions. For finite S-parameters at dc, it is assumed that a dc term in the denominator is not zero and is set equal to one:

$$F_{11}(\omega_i) = 1 - S_{11} = \frac{\sum_{k=0}^{m} b_k^{11} (j\omega_i)^k}{1 + \sum_{k=1}^{n} a_k (j\omega_i)^k} \tag{14}$$

$$F_{12}(\omega_i) = S_{12} = \frac{\sum_{k=0}^{m} b_k^{12} (j\omega_i)^k}{1 + \sum_{k=1}^{n} a_k (j\omega_i)^k} \tag{15}$$

$$F_{21}(\omega_i) = S_{21}$$

$$= \frac{\sum_{k=0}^{m} b_k^{21}(j\omega_i)^k}{1+\sum_{k=1}^{n} a_k(j\omega_i)^k} \tag{16}$$

$$F_{22}(j\omega_i) = 1 - S_{22} = \frac{\sum_{k=0}^{m} b_k^{22}(j\omega_i)^k}{1+\sum_{k=1}^{n} a_k(j\omega_i)^k} \tag{17}$$

where m and n are orders of the numerators and the denominator, respectively.

In (14) and (17), $1-S_{11}$ and $1-S_{22}$ are used instead of S_{11} and S_{22} to reduce order of the numerator. The error function is

$$\epsilon = \sum_{i=1}^{N} \sum_{p,q=1}^{2} \left| \left\{ 1+\sum_{k=1}^{n} a_k(j\omega_i)^k \right\} \hat{F}_{pq}(\omega_i) - \sum_{k=0}^{m} b_k^{pq}(j\omega_i)^k \right|^2. \tag{18}$$

Partial differentiation of ϵ with respect to a_k and b_k gives the following simultaneous equations:

$$\begin{bmatrix} A & B_{11} & B_{12} & B_{21} & B_{22} \\ C_{11} & D & 0 & 0 & 0 \\ C_{12} & 0 & D & 0 & 0 \\ C_{21} & 0 & 0 & D & 0 \\ C_{22} & 0 & 0 & 0 & D \end{bmatrix} \begin{bmatrix} a \\ b^{11} \\ b^{12} \\ b^{21} \\ b^{22} \end{bmatrix} = \begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix}. \tag{19}$$

The elements in the partial matrixes are

$$\begin{aligned} [A]_{pq} &= \sum_{i=1}^{N} \sum_{k,l=1}^{2} \{(j\omega_i)^p(-j\omega_i)^q + (-j\omega_i)^p(j\omega_i)^q\} \hat{F}_{kl}(\omega_i)\hat{F}_{kl}^*(\omega_i) \\ [B_{kl}]_{pq} &= -\sum_{i=1}^{N} \{(j\omega_i)^p(-j\omega_i)^q \hat{F}_{kl}(\omega_i) + (-j\omega_i)^p(j\omega_i)^q \hat{F}_{kl}^*(\omega_i)\} \\ [C_{kl}]_{pq} &= -\sum_{i=1}^{N} \{(j\omega_i)^p(-j\omega_i)^q \hat{F}_{kl}^*(\omega_i) + (-j\omega_i)^p(j\omega_i)^q \hat{F}_{kl}(\omega_i)\} \\ [D]_{pq} &= \sum_{i=1}^{N} \{(j\omega_i)^p(-j\omega_i)^q + (-j\omega_i)^p(j\omega_i)^q\} \end{aligned} \tag{20a}$$

$$[c_0]_p = -\sum_{i=1}^{N} \{(j\omega_i)^p + (-j\omega_i)^p\} \hat{F}_{kl}(j\omega_i)\hat{F}_{kl}^*(j\omega_i). \tag{20b}$$

Substituting (1) to (5) into (6) shows that order of the denominator is higher than these of the numerators by two. Our experience of fitting the measured S-parameters suggests the orders of m and n are 5 and 7, respectively.

V. Determination of the Element Values

The set of rational functions not only describes the FET behavior, it also gives information on the initial values for the iterative determination of the equivalent circuit. Extraction of the initial values is described below.

Substituting (4) into (6), we get

$$(1-S_{11}) - S_{12} = \frac{2Z_0\{Y_{11}+Y_{12}+(Z_0+Z_d)\Delta\}}{P\Delta} \tag{21}$$

$$S_{12} = \frac{2Z_0\{Z_s\Delta - Y_{12}\}}{P\Delta} \tag{22}$$

$$S_{12} - S_{21} = \frac{2Z_0(Y_{21}-Y_{12})}{P\Delta} \tag{23}$$

$$(1-S_{22}) - S_{12} = \frac{2Z_0\{Y_{22}+Y_{12}+(Z_0+Z_g)\Delta\}}{P\Delta} \tag{24}$$

Equations (21) to (24) can be rewritten using rational functions as follows:

$$(1-S_{11}) - S_{12} = \frac{\sum_{k=0}^{m} (b_k^{11} - b_k^{12})(j\omega_i)^k}{1+\sum_{k=1}^{n} a_k(j\omega_i)^k} \tag{25}$$

$$S_{12} = \frac{\sum_{k=0}^{m} b_k^{12}(j\omega_i)^k}{1+\sum_{k=1}^{n} a_k(j\omega_i)^k} \tag{26}$$

$$S_{12} - S_{21} = \frac{\sum_{k=0}^{m} (b_k^{12} - b_k^{21})(j\omega_i)^k}{1+\sum_{k=1}^{n} a_k(j\omega_i)^k} \tag{27}$$

$$(1-S_{22}) - S_{12} = \frac{\sum_{k=0}^{m} (b_k^{22} - b_k^{12})(j\omega_i)^k}{1+\sum_{k=1}^{n} a_k(j\omega_i)^k} \tag{28}$$

From the lower-order terms of the numerators of (25) to (28), the initial values of all intrinsic elements in Fig. 1 are extracted. The extraction procedure is described in the Appendix, and the results are summarized in (29) at the bottom of the next page and in (30)–(36) below:

$$\tau = \frac{b_1^{21} - b_1^{12}}{b_0^{21} - b_0^{12}} - \tau_i \tag{30}$$

$$g_m = \frac{b_0^{21} - b_0^{12}}{\{2 - (b_0^{22} - b_0^{12})\} Z_0} \tag{31}$$

$$R_{ds} = \frac{b_0^{21} - b_0^{12}}{b_0^{22} - b_0^{12}} \frac{1}{g_m} \tag{32}$$

$$C_{gd} = \frac{1 + \frac{Z_0}{R_{ds}}}{2Z_0} b_1^{12} \tag{33}$$

$$C_{gs} = \frac{b_1^{11} - b_1^{12}}{2Z_0} - \frac{2Z_0C_{gd}\left(g_m + \frac{1}{R_{ds}}\right)}{1 + \frac{Z_0}{R_{ds}}} \quad (34)$$

$$R_i = \frac{\tau_i}{C_{gs}} \quad (35)$$

$$C_{ds} = \left(1 + \frac{Z_0}{R_{ds}}\right)\frac{b_1^{22} - b_1^{12}}{2Z_0} - Z_0\left(g_mC_{gd} + \frac{C_{gd} + C_{gs}}{R_{ds}}\right). \quad (36)$$

In (29), measurement error sometimes results in a negative square root, producing three different cases for τ_i.

Since higher order coefficients of ω are smaller, these contain relatively larger errors. As a result, it is difficult to extract extrinsic elements which correspond to higher order coefficients. So, an iterative algorithm is used. All the initial values of extrinsic elements are set equal to zero, and the gradients of the S-parameters for all elements are calculated, i.e.,

$$\frac{\partial S_{pq}}{\partial R_g}\frac{\partial S_{pq}}{\partial L_g}\ldots \text{etc.}$$

All element values are iteratively updated by the following equation to minimize deviation ϵ,

$$\epsilon = \sum_{i=1}^{N}\sum_{p,q=1}^{2}|S_{pq}(j\omega_i) - \hat{S}_{pq}(j\omega_i)|^2 \quad (37)$$

where S_{pq} is the calculated S-parameters using the equivalent circuit and $\hat{S}_{pq}$ is the measured S-parameters.

The Newton–Raphson method is used in the iteration, assuming a damping factor of 0.5. Convergence is obtained within ten iterations.

In conventional methods, the selection of the initial values sometimes affects the final results, and this has been troublesome for those with limited technical background. In this method, the initial values are determined automatically from the coefficients of the rational functions.

VI. Measurement

For verification, three GaAs MESFET's with different gate widths were measured. Each FET had a gate length of 0.25 μm. Fig. 2 compares the calculated and measured S-parameter values for the 400-μm gate width. Squares denote measured values, crosses values calculated by rational functions, and solid lines those obtained from the equivalent circuit. Differences are almost indistinguishable. Figs. 3 to 6 show the real and imaginary parts of the S-parameters. Once again, the differences are negligible. The rms errors, ϵ_{pq}, between S-parameters measured and calculated from the equivalent circuit, were calculated from

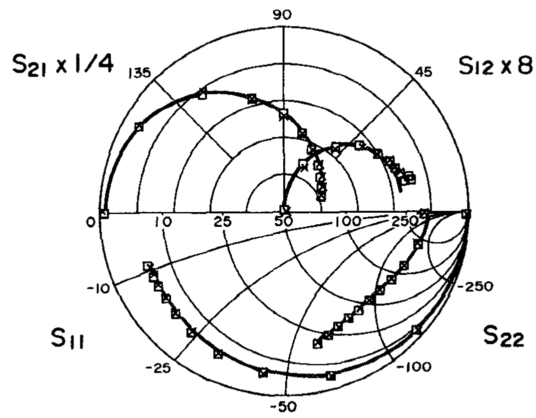

Fig. 2. S-parameters of 400-μm GaAs MESFET. Frequency: 0.045 to 26.5 GHz. Bias: $V_{ds} = 8$ V, $I_{ds} = 0.6\,I_{dss}$. Squares: measured values. Crosses: calculated values using rational functions. Lines: calculated values using equivalent circuit.

$$\epsilon_{pq} = \sqrt{\frac{\sum_{i=1}^{N}|S_{pq}(\omega_i) - \hat{S}_{pq}(\omega_i)|^2}{\sum_{i=1}^{N}|\hat{S}_{pq}(\omega_i)|^2}} \quad (38)$$

where $\epsilon_{11} = 1.17\%, \epsilon_{12} = 3.97\%, \epsilon_{21} = 1.11\%$, and $\epsilon_{22} = 1.11\%$.

The comparison between the measured and calculated S-parameters of the MESFET's having the gate widths of 200 and 600 μm are also shown in the Figs. 7 and 8. The obtained equivalent circuit parameters of the different gate-width MESFET's are shown in Table I.

The intrinsic elements obey to the scale law. This suggests uniformity in the fabrication process. However, the extrinsic elements deviate from the scale law. These FET's are composed of unit FET's with a gate width of 50 μm. Unfortunately, the unit FET's geometrical layout is not the same for FET's with 200, 400, and 600 μm gates. Scale law deviation may be a result of the layout difference. Present FET structures are rather complicated and it is difficult to explain this deviation. We are now discussing the problem with people

$$\tau_i = \begin{cases} \sqrt{\left(\frac{b_1^{21} - b_1^{12}}{b_0^{21} - b_0^{12}}\right)^2 - \frac{2(b_2^{21} - b_2^{12})}{b_0^{21} - b_0^{12}}} & \text{if } \left(\frac{b_1^{21} - b_1^{12}}{b_0^{21} - b_0^{12}}\right)^2 - \frac{2(b_2^{21} - b_2^{12})}{b_0^{21} - b_0^{12}} > 0 \\ 0 & \text{if } \left(\frac{b_1^{21} - b_1^{12}}{b_0^{21} - b_0^{12}}\right)^2 - \frac{2(b_2^{21} - b_2^{12})}{b_0^{21} - b_0^{12}} < 0, \frac{b_1^{21} - b_1^{12}}{b_0^{21} - b_0^{12}} < 0 \\ \frac{b_1^{21} - b_1^{12}}{b_0^{21} - b_0^{12}} & \text{if } \left(\frac{b_1^{21} - b_1^{12}}{b_0^{21} - b_0^{12}}\right)^2 - \frac{2(b_2^{21} - b_2^{12})}{b_0^{21} - b_0^{12}} < 0, \frac{b_1^{21} - b_1^{12}}{b_0^{21} - b_0^{12}} > 0 \end{cases} \quad (29)$$

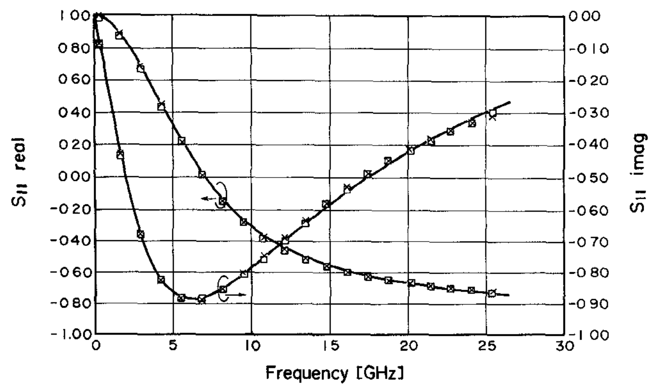

Fig. 3. Real and Imaginary parts of S_{11} (W_g = 400 μm). Squares: measured values. Crosses: calculated values using rational functions. Lines: calculated values using equivalent circuit.

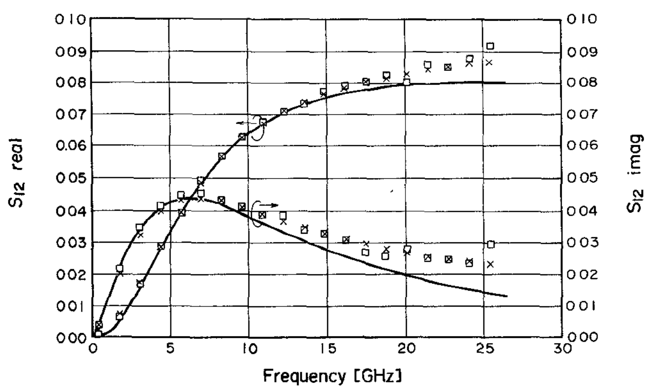

Fig. 4. Real and Imaginary parts of S_{12} (W_g = 400 μm). Squares: measured values. Crosses: calculated values using rational functions. Lines: calculated values using equivalent circuit.

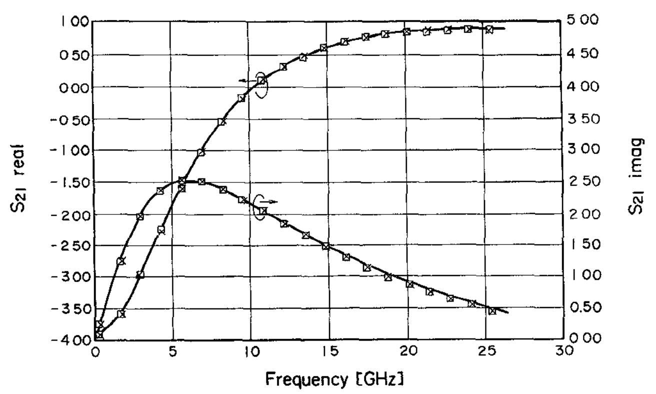

Fig. 5. Real and Imaginary parts of S_{21} (W_g = 400 μm). Squares: measured values. Crosses: calculated values using rational functions. Lines: calculated values using equivalent circuit.

at the compound semiconductor laboratory and attempting to design a new FET structure which is suitable for examining the scale law.

VII. Conclusion

We have proposed a new method for describing the behavior of microwave devices by using a set of rational functions of angular frequency ω. Applying this technique to the S-parameters of GaAs MESFET's having different gate widths, we obtained excellent agreement with measured values. The set of rational functions gives adequate information for the circuit design and uses significantly less memory than tabulated

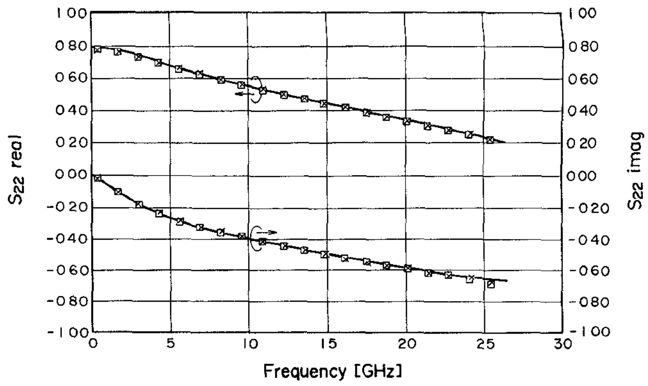

Fig. 6. Real and Imaginary parts of S_{22} (W_g = 400 μm). Squares: measured values. Crosses: calculated values using rational functions. Lines: calculated values using equivalent circuit.

TABLE I
Equivalent Circuit Elements

Element	Gate width (μm)		
	200	400	600
Rg (Ω)	1.633	0.672	0.171
Rs (Ω)	3.261	1.300	0.729
Rd (Ω)	14 899	2.108	1.466
Lg (pH)	33.261	32.508	30.637
Ls (pH)	1.710	1.927	3.463
Ld (pH)	6.886	6 100	11 049
Ri (Ω)	5.984	4.078	3 161
Rds (Ω)	911.459	395.670	252.01
gm (mS)	23.877	46.651	72 145
τ (ps)	4.949	5.821	6 099
Cgs (pF)	0.192	0.388	0.592
Cgd (pF)	0.011	0.022	0.033
Cds (pF)	0.036	0.071	0.107

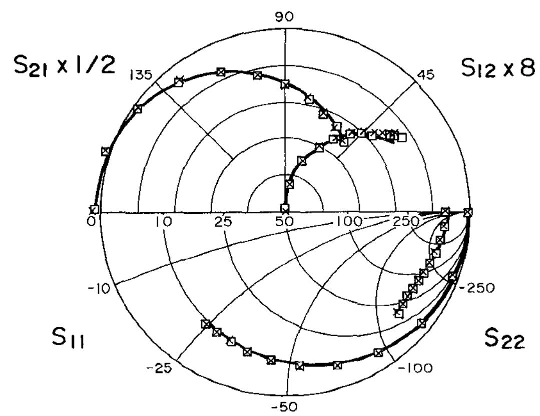

Fig. 7. S-parameters of 200-μm GaAs MESFET. Frequency: 0.045 to 26.5 GHz. Bias: V_{ds} = 8 V, I_{ds} = 0.6 I_{dss}. Squares: measured values. Crosses: calculated values using rational functions. Lines: calculated values using equivalent circuit.

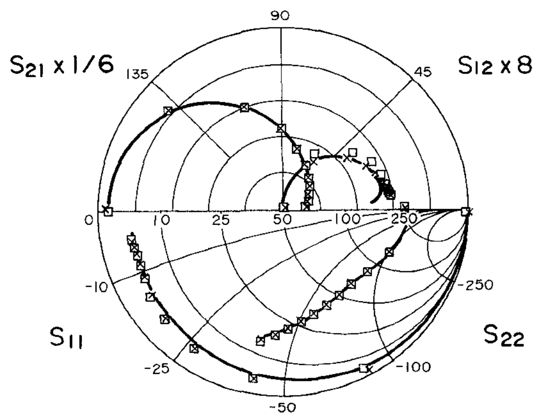

Fig. 8. S-parameters of 600-μm GaAs MESFET. Frequency: 0.045 to 26.5 GHz. Bias: $V_{ds} = 8$ V, $I_{ds} = 0.6\, I_{dss}$. Squares: measured values. Crosses: calculated values using rational functions. Lines: calculated values using equivalent circuit.

S-parameters.

We have shown a way of determining the equivalent circuit from the rational functions. The effect of initial-value selection is eliminated by determining initial values from the coefficients of the rational functions. The proposed method also requires no additional measurements to determine the extrinsic elements. The equivalent circuit consistently enables the implementation of our procedure on the currently available microwave CAD software.

Appendix

Substituting (1) and (2) into (21) to (24), and setting them equal to (25) to (28) respectively, we get

$$\frac{2Z_0\{j\omega C_{gs} + (1+j\omega\tau_i)(Z_0+Z_d)\Delta\}}{(1+j\omega\tau_i)P\Delta} = \frac{\sum_{k=0}^{m}(b_k^{11}-b_k^{12})(j\omega_i)^k}{1+\sum_{k=1}^{n}a_k(j\omega_i)^k}$$

$$\frac{2Z_0\{j\omega C_{gd} + Z_s\Delta\}(1+j\omega\tau_i)}{(1+j\omega\tau_i)P\Delta} = \frac{\sum_{k=0}^{m}b_k^{12}(j\omega_i)^k}{1+\sum_{k=1}^{n}a_k(j\omega_i)^k}$$

$$\frac{2Z_0 g_m e^{-j\omega\tau}(1+j\omega\tau_i)}{(1+j\omega\tau_i)P\Delta} = \frac{\sum_{k=0}^{m}(b_k^{21}-b_k^{12})(j\omega_i)^k}{1+\sum_{k=1}^{n}a_k(j\omega_i)^k}$$

$$\frac{2Z_0\left\{\frac{1}{R_{ds}} + j\omega C_{ds} + (Z_0+Z_g)\Delta\right\}(1+j\omega\tau_i)}{(1+j\omega\tau_i)P\Delta} = \frac{\sum_{k=0}^{m}(b_k^{22}-b_k^{12})(j\omega_i)^k}{1+\sum_{k=1}^{n}a_k(j\omega_i)^k}$$

where Δ is the determinant of $Y_{\text{intrinsic}}$, which is shown at the bottom of the page and

$$\tau_i = C_{gs}R_i.$$

The term in the denominator with the smallest order which normalizes all coefficients is obtained from

$$(1+j\omega\tau_i)P\Delta|_{\omega=0} = 1 + \frac{Z_0+R_d+R_s}{R_{ds}} + g_m R_s.$$

Then the coefficients of each of the rational functions' numerator are derived as follows:

$$\frac{2Z_0\{C_{gs}+(Z_0+R_d)S\}}{T} = (b_1^{11}-b_1^{12})$$

$$\frac{2Z_0\{C_{gd}+R_sS\}}{T} = b_1^{12}$$

$$\frac{2Z_0 g_m}{T} = (b_0^{21}-b_0^{12})$$

$$\frac{2Z_0 g_m(\tau_i-\tau)}{T} = (b_1^{21}-b_1^{12})$$

$$\frac{2Z_0 g_m\tau\left(\tau_i-\frac{\tau}{2}\right)}{T} = (b_2^{21}-b_2^{12})$$

$$2Z_0\frac{\frac{1}{R_{ds}}}{T} = (b_0^{22}-b_0^{12})$$

$$\frac{2Z_0\{C_{ds}+(Z_0+R_g)S\}}{T} = (b_1^{22}-b_1^{12})$$

where

$$S = \frac{C_{gs}+C_{gd}}{R_{ds}} + g_m C_{gd}$$

$$T = 1 + \frac{Z_0+R_d+R_s}{R_s} + g_m R_s.$$

$$\Delta = \frac{j\omega C_{gs}\left(\frac{1}{R_{ds}} + j\omega C_{ds}\right) + j\omega C_{gd}\left\{j\omega C_{gs} + (1+j\omega\tau_i)\left(g_m e^{-j\omega\tau} + \frac{1}{R_{ds}} + j\omega C_{ds}\right)\right\}}{1+j\omega\tau_i}.$$

Using the third row of the above equations:

$$\tau_i - \tau = \frac{b_1^{21} - b_1^{12}}{b_0^{21} - b_0^{12}}$$

$$(\tau - \tau_i)^2 - \tau^2 = -\frac{2(b_2^{21} - b_2^{12})}{b_0^{21} - b_0^{12}}$$

The ratio of $(b_0^{21} - b_0^{12})$ to $(b_0^{22} - b_0^{12})$ is

$$g_m R_{ds} = \frac{b_0^{21} - b_0^{12}}{b_0^{22} - b_0^{12}}$$

and, realizing that Z_0 is always large in comparison to R_s and R_d,

$$\frac{2Z_0 g_m}{1 + Z_0 g_m \frac{b_0^{22} - b_0^{12}}{b_0^{21} - b_0^{12}}} \approx (b_0^{21} - b_0^{12})$$

$$ie \quad g_m = \frac{b_0^{21} - b_0^{12}}{\{2 - (b_0^{22} - b_0^{12})\} = sZ_0}$$

In the same way, we can determine C_{gd} from b_1^{12}, C_{gs} from $(b_1^{11} - b_1^{12})$ and C_{ds} from $(b_1^{22} - b_1^{12})$.

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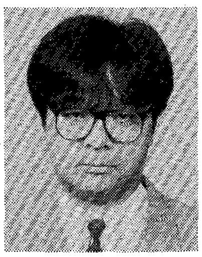

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